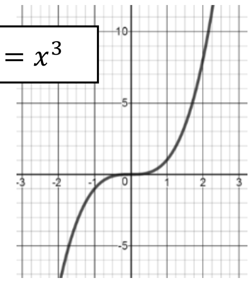
AP Calculus

How Do You Link Riemann Sum to Definite Integrals: The Graphic Novel!

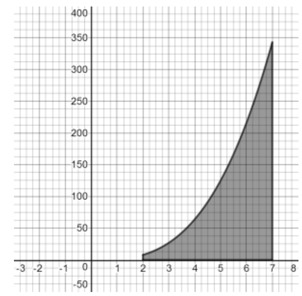
$\int_2^7 x^3 dx$

What is this?
...Mommy?

$f(x) = x^3$

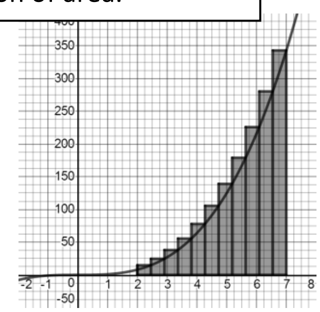


= The exact area between
$x = 2,\ \ x = 7,$
$y = 0,\ and\ \ \ y = x^3$



= The sum of infinite rectangles under the curve of $f(x)=x^3$ to get the exact area.

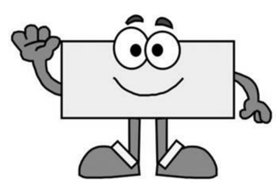
Increase the number of rectangles to get more and more precise estimation of area.



n=total # of rectangles

= $\lim_{n\to\infty} \sum_{k=1}^{n}(\text{Area of each rectangle under f(x)}).$

k=which rectangle are you currently adding to your sum.

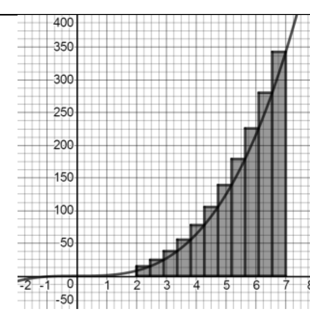


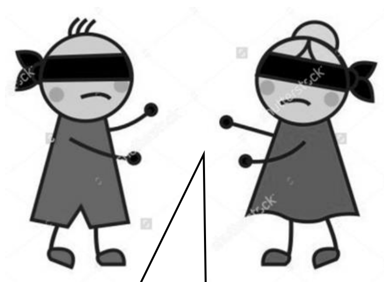
If only I knew how to find the area of a rectangle…

= $\lim_{n\to\infty} \sum_{k=1}^{n}(\text{height of rectangle})(\text{width of rectangle}).$

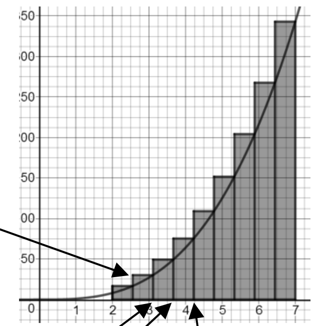
= $\lim_{n\to\infty} \sum_{k=1}^{n}(current\ x\ value)^3 \left(\frac{5}{n}\right)$

If we have n rectangles in 2 ≤ x ≤ 7, then each rectangle is $\frac{7-2}{n} = \frac{5}{n}$ wide.





Hmmmm, where is my “current rectangle” if I can’t really see each rectangle???



$x = 2 + \frac{5}{n}$

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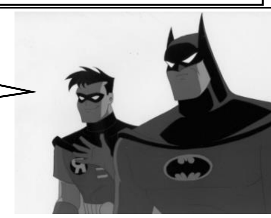
$= \lim_{n\to\infty} \sum_{k=1}^{n} \left(2 + k\frac{5}{n}\right)^3 \left(\frac{5}{n}\right)$

I start at 2 and then add $\frac{5}{n}$ k times …. For however many rectangles I am moving right!

$\int_2^7 x^3 dx = \lim_{n\to\infty} \sum_{k=1}^{n} \left(2 + k\frac{5}{n}\right)^3 \left(\frac{5}{n}\right)$

Holy Transitive Property, Batman! I think we got it!



Exact area under the curve.

Need infinite rectangles to be exact.

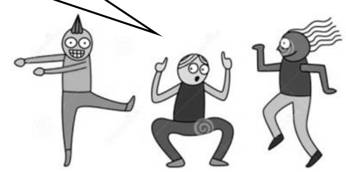
$\int_2^7 x^3 dx = \lim_{n\to\infty} \sum_{k=1}^{n} \left(2 + k\frac{5}{n}\right)^3 \left(\frac{5}{n}\right)$

Width of every rectangle

k is the current rectangle.

Height of the k^{th} rectangle from the left when you have n rectangles.

Break it down!



Homework:

Find the exact Riemann Equation that represents the following exact areas.

1. $\int_5^{11} x^2 dx$
2. $\int_{-2}^{4} 3x^4\, dx$
3. $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin x\, dx$
4. $\int_{-10}^{-3} x^8 dx$