Equations with Radical Expressions
Algebra 2

Goals:
1. Simplify expressions involving rational expressions. (1.01)
2. Translate among graphic, algebraic, and verbal representations of relations. (3.02)
3. Use quadratic equations and inequalities to solve problems. Solve by: graphing, factoring. (3.05)
4. Use systems of two equations to solve problems. Solve by graphing. (3.12)
5. Use equations which contain radical expressions to solve problems. Solve by: graphing, factoring, or using properties of equality. (3.11)

Materials Needed:
1. One copy of the handout for each student
2. Graphing calculator
3. Paper and pencil for algebraic solutions and note taking.

Activity 1:

Students took the following data:

Length of pendulum in cm	18	16	24	32	39	45	53	63
Period of swing in secs	1.4204	1.378	1.6604	1.9122	2.1006	2.2544	2.4068	2.6152

The **length** of the pendulum measures the length of the string and weight that form the pendulum. The **period** of the swing measures the length of time it takes the pendulum to go from one side to the other side and back. These values were found by measuring 10 periods and finding the average.

Have students put the data in their calculator and graph the scatter plot. Think about domain issues to get a good graphing window. The equation $y = \frac{1}{3}\sqrt{x}$ serves as a good model for the data (not found through statistical process), where y represents the time for one cycle and x represents the length of the pendulum. Superimpose this equation over the data. Ask students whether or not this is a good fit. They should be able to substantiate their claim. The model is a fairly good model for the data. The scatter plot of residuals is random and the values of residuals are relatively small.

Suppose we want our pendulum to behave like a clock. Every time the pendulum ticks, a second passes. This means each period should take two seconds. How long should we make our string? We need to solve the equation $2 = \frac{1}{3}\sqrt{x}$:

$$2 = \frac{1}{3}\sqrt{x}$$
$$6 = \sqrt{x}$$
$$6^2 = \left(\sqrt{x}\right)^2$$
$$36 = x$$

Our string should be 36 cm long in order to have a period take two seconds.

Now suppose we have a string that is 21 cm long. How long will the period of our pendulum be? We need to solve the equation $y = \frac{1}{3}\sqrt{21}$:

$$y = \frac{1}{3}\sqrt{21} \doteq 1.53$$

The period of our pendulum would be approximately 1.53 seconds (exactly $\frac{1}{3}\sqrt{21}$ seconds).

Activity 2:
Students will be working with equations that contain radicals. We will use the graphing calculator to find solutions with discussion of restrictions in the domain and range of functions containing radicals. Also, students will discover graphically that squaring both sides of the equation to eliminate a radical produces the same root(s) and can introduce extraneous root(s)

1. Solve $\sqrt{6x+4} = x+1$ for x. (This is from the Algebra II Indicators prepared by NC Department of Public Instruction.)
   - Using the graphing calculator, we can graph two functions $Y1 = \sqrt{6x+4}$ and $Y2 = x+1$ to find the point(s) of intersection. These two graphs intersect at approximately $(-0.65, 0.35)$ and $(4.65, 5.65)$. Discuss issues of domains of the two functions while looking at the graphs. A good window for this graphing activity is $-10 \le x \le 10, -10 \le y \le 10$.
   - If this problem is done algebraically, the first step would be to square both sides of the equation.

$$\sqrt{6x+4} = x+1$$
$$6x+4 = (x+1)^2$$
$$6x+4 = x^2 + 2x + 1$$

If the graphs done above with $Y1 = \sqrt{6x+4}$ and $Y2 = x+1$ are done on the same coordinate system as $Y3 = 6x+4$ and $Y4 = x^2 + 2x + 1$, the points of intersection of the two sets of graphs will have the same $x-$values and will produce the same solutions. Window must be changed for $-10 \le y \le 40$. There may need to be some discussion of why the y-values are different. Does that always work – i.e., looking at the intersection of a different pair of “related” functions?

If we continue this solution and use the quadratic formula,

$$6x+4=x^2+2x+1$$
$$0=x^2-4x-3$$
$$x=\frac{4\pm\sqrt{(-4)^2-4(1)(-3)}}{2(1)}$$
$$x=\frac{4\pm\sqrt{28}}{2}$$
$$x=2\pm\sqrt{7}$$

2. Solve $\sqrt{2x+1}=x-5$ for x.
   - If we follow the procedures of the last problem, the graphs of $Y1=\sqrt{2x+1}$ and $Y2=x-5$ show a point of intersection at $(9.46, 4.46)$. (Note: since Y1 has a restricted domain, the initial x-value must be appropriate for the domain when beginning the intersect procedure on the calculator.) This has only one result, while the previous question had two. Suggested window: $-2\le x\le 10; -2\le y\le 10$.
   - If we continue with the algebraic investigation of this equation, we would again square both sides:
   $$\sqrt{2x+1}=x-5$$
   $$2x+1=x^2-10x+25$$
   If we graph the Y1 and Y2 from above with $Y3=2x+1$ and $Y4=x^2-10x+25$, the graphs of Y3 and Y4 show two points of intersection. One has the same $x-$value as above $x=9.46$, but the second point shows $x=2.54$. Notice this is a value of x that will produce a negative value for $x-5$. This cannot equal the value of $\sqrt{2x+1}$ which is non-negative. Here is the case that squaring both sides of the equation introduces a root that is not possible. (The graph of $y=-\sqrt{2x+1}$ intersects with $y=x-5$ at the “impossible” root.)
   Continuing the algebraic solution:
   $$2x+1=x^2-10x+25$$
   $$0=x^2-12x+24$$
   $$x=\frac{-(-12)\pm\sqrt{(-12)^2-4(1)(24)}}{2(1)}$$
   $$x=\frac{12\pm\sqrt{48}}{2}$$
   $$x=6\pm2\sqrt{3}$$
   When $x=6-2\sqrt{3}$, the value of $x-5$ is negative and is not a possible solution.

Follow Up Activities

1. For what values of x is $4+\sqrt{4x-5}\le 10$?
   - We need to first consider this as an equality rather than an inequality in order to find the point(s) of intersection. Beyond realizing this, we need to decide whether to consider the problem to $4+\sqrt{4x-5}=10$ or $\sqrt{4x-5}\le 6$. The result should be the same. Algebraically, the second form is simpler to work with. The graphs of $Y1=\sqrt{4x-5}$ and $Y2=6$ show a point of intersection at $(10.25,6)$. There is only one point of intersection. Look at a graphs to decide about the inequality. Graphically, the inequality $\sqrt{4x-5}\le 6$ is asking us where the graph of $Y1=\sqrt{4x-5}$ is *lower* than the graph of $Y2=6$. This occurs from the point at which the square root function is first defined $\left(4x-5\ge 0 \text{ or } x\ge \frac{5}{4}\right)$ to the point of intersection $(10.25,6)$. So, we see the inequality is true for $1.25\le x\le 10.25$.
   - If we continue the algebraic investigation of the equation, we would begin by squaring both sides of the equation:

$$\sqrt{4x-5}=6$$
$$\left(4x-5\right)=6^2$$
$$4x=41$$
$$x=\frac{41}{4}=10.25$$

   We then need to make a decision about the inequality. At $x=10.25$ they are equal, so either $x\ge 10.25$ or $1.25\le x\le 10.25$ could represent the correct interval. Choose test values to help decide.

   If $x=2$ (a value between 1.25 and 10.25) we have $\sqrt{4(2)-5}\le 6$ which *is* a true statement, so $1.25\le x\le 10.25$ *is* the correct interval.
   Just to check and make sure, go ahead and check the other part of the interval. If $x=12$ (a value greater than 10.25) we have $\sqrt{4(12)-5}=\sqrt{43}\le 6$ which *is not* a true statement, so $x\ge 10.25$ *is not* the correct interval.

2. Is it true that $\sqrt{x^2+25}=x+5$?
   - From a graphical point of view, the graphs of $Y1=\sqrt{x^2+25}$ and $Y2=x+5$ only intersect once, so it is not true that $\sqrt{x^2+25}=x+5$ all the time. The point of intersection is found to be $(0,0)$.
   - Algebraically we can confirm this:

$$\sqrt{x^2+25}=x+5$$
$$\left(\sqrt{x^2+25}\right)^2=\left(x+5\right)^2$$
$$x^2+25=x^2+10x+25$$
$$10x=0$$
$$x=0$$

Student Handout
Algebra 2
Equations with Radical Expressions

1. Students took the following data:

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a. Put the data in your calculator and graph the scatter plot.

b. The equation $y = \frac{1}{3}\sqrt{x}$ serves as a good model for the data (not found through statistical process), where y represents the time for one cycle and x represents the length of the pendulum. Superimpose this equation over the data. Is this curve a good fit for the data? Provide support for your answer.

c. Suppose we want our pendulum to behave like a clock and keep relatively accurate time. This means each period should take two seconds. How long should we make our string?

d. Suppose we have a string that is 21 cm long. How long will the period of our pendulum be?

2. Solve $\sqrt{6x+4} = x+1$ for x.

3. Solve $\sqrt{2x+1} = x-5$ for x.

Follow Up Problems
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1. For what values of x is $4+\sqrt{4x-5}\leq 10$?

2. Is it true that $\sqrt{x^2+25}=x+5$?